The main object of this thesis is the theoretical and numerical analysis of **box schemes**. This class of schemes, has been introduced by H.B. Keller in 1971 for parabolic problems. In the case of elliptic problems, the basic principle is to average the two continuous equations (conservation and flux) given by the mixed form of the problem, onto the boxes of the mesh. Box schemes belong to the category of so-called mixed Petrov-Galerkin finite volume methods. Indeed, the approximation is performed on the mixed form of the problem, with a pair of trial spaces different from the pair of test spaces. The trial spaces (for u and ∇u) are of finite element type and the test spaces are of Galerkin-discontinuous type. The selection of the different spaces functions (trial and test) is difficult, because they have to satisfy the compatibility Babǔska-Brezzi condition. However, contrary to other schemes, the method requires an unique mesh. In most of the cases, the scheme is equivalent to a variational formulation in the principal unknow (u) and a local reconstruction of the flux (∇u). Firstly, I studied the bidimensional mixed form of the Poisson problem with a box scheme on triangular or quadrangular meshes. Stability results and error estimates are given using the finite element theory. A numerical study on several test cases (Matlab code) completes our theoretical results.

As part of the research group MoMaS for deep ground repositories of radioactive wastes, the potential interest of box schemes for unstationary convection-diffusion problems has been tested. A box scheme has been designed for the 1D equation. Two kinds of upwinding are introduced, each one being designed to cure the two classical oscillations sources present in the approximation of convective-diffusion equations. The generalization to the bidimensional case is perfomed using an ADI-like method (Alternating Direction Implicit).